

Bargaining with Deadlines and Private Information[†]

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We study dynamic bargaining with private information and a deadline. As commitment power disappears, there is a clear “deadline effect.” That is, trade takes place smoothly before the deadline and with an atom right at the deadline. Prices, timing of trade, and the deadline effect respond to the consequences of not reaching an agreement. Bleaker disagreement options lead to more trade and proportionally more of the agreements taking place on the verge of the deadline. Time to deadline can affect the overall efficiency of the equilibrium nonmonotonically. For intermediate deadlines, efficiency is improved if agents face bleaker prospects after deadline. (JEL C78, D82)

In this paper, we study dynamic bargaining with private information in the presence of a deadline. Many negotiations have a preset deadline by which an agreement must be reached. For example, with a known trial date looming ahead, parties engage in pretrial negotiations. Before international summits, countries bargain over the terms of the accords to be signed at the summit. Broadcasters selling advertising space for some live event have until the event takes place to reach an agreement with the advertisers. Negotiations to renew labor contracts have until the expiration date of the current contract or the preset strike date if conflicts are to be avoided.

Financial considerations might also act as an effective deadline. Countries that have large debt repayments ahead of them bargain with international agencies such as the IMF for financing that would help them avoid default.¹ Private companies also face refinancing deadlines or deadlines to obtain financing in order to be able to invest in a given venture.

Finally, negotiations can be affected by regulatory deadlines. For example, to take advantage of the home buyer credit program, buyers and sellers of homes had to close their transactions by a given deadline to qualify for the subsidy.

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¹For example, the current negotiations between Greece, the EU, and IMF are carried under the looming refinancing needs due to loans maturing. “Greece must refinance 54 billion euros in debt in 2010, with a crunch in the second quarter as more than 20 billion euros becomes due.” See David Cutler, “Greece’s debt crisis,” Reuters, April 16, 2010, accessed September 12, 2013. <http://www.reuters.com/article/idUSTRE63F2ZR20100416>.

Empirical literature has documented that a large fraction of agreements are reached in the “eleventh hour” that is at or very close to the deadline. For example, Cramton and Tracy (1992) study a sample of 5,002 labor contract negotiations involving large bargaining units and they claim a “clear ‘deadline effect’ exists in the data” since 31 percent of agreements are reached on the deadline.² Williams (1983), in a sample of civil cases from Arizona, has found that 70 percent of the cases were settled in the last 30 days before trial and 13 percent were settled on the day of the trial. Such strong deadline effects have also been observed in experimental studies (see, for example, Roth, Murnighan, and Schoumaker 1988; or Güth, Levati, and Maciejovsky 2005).

What determines whether parties will reach an agreement before the deadline, at the deadline (on the morning of the trial date or in the wee hours of the night before the labor contract expires) or not at all? How do postdeadline payoffs affect the division of surplus and the timing of agreement? What else affects these two key aspects of the dynamic outcomes?

We answer these questions in a model of bargaining with a deadline based on a classic paper by Sobel and Takahashi (1983)—henceforth, ST.³ In the model, a seller makes an offer that can be either accepted or refused. If rejected, the process continues until a deadline is reached. The buyer has private information about the value of the good for sale (i.e., we have one-sided private information). Both parties prefer earlier trade, this is modeled via impatience (discounting costs) to realize the surplus from trade.

We add to the ST model in two ways. First, ST consider only the case that if trade does not take place by the deadline, the trade opportunity and all surplus is lost. Since we are interested in how postdeadline outcomes affect the size of the deadline effect and the division of surplus, the amount of remaining surplus and its split after the deadline are the key variables in our model.⁴ Second, more technically, we take the noncommitment limit (i.e., taking the time between periods to zero) and show that the limit of equilibria is very simple. Moreover, it allows for a clear definition of the “deadline effect” as in the limit there is smooth trade (probability flow) before the deadline and an atom (probability mass) of trade at the deadline.⁵ Additionally, prices in the limit have a natural economic interpretation related to the intuition for the Coase conjecture (despite trade being inefficient). The tractability of the continuous time limit is somewhat unexpected because non-stationary models (with time until the deadline elapses) are usually much more difficult to analyze, as can be seen in discrete time by comparing ST to Stokey (1981) and Bulow (1982).

²They interpret agreements reached even a day after the contract expiration as reached “on” the deadline.

³There are other bargaining models with deadlines but they are quite different. See, for example, Ma and Manove (1993) and Fershtman and Seidmann (1993).

⁴Admittedly, each of the bargaining environments provided as examples above has idiosyncratic and potentially important details that would affect the way negotiations are carried forward. Our model abstracts from many of those details, yet it is rich enough to capture the effect of deadlines and the consequence of not reaching an agreement on the bargaining outcome. The disagreement payoff can also be thought of as the expected payoffs the agents would get from the continuation game that would start at $T + 1$. For example, if the private information is revealed at $T + 1$ (possibly at some cost) and then the players bargain with full information.

⁵Away from the limit there is a mass of trade in each period and hence the “deadline effect” is less evident.

Dynamic bargaining with asymmetric information has been studied extensively since the classic results on the Coase conjecture by Stokey (1981); Bulow (1982); Fudenberg, Levine, and Tirole (1985)—henceforth, FLT; and Gul, Sonnenschein, and Wilson (1986)—henceforth, GSW.⁶ In such games, the seller becomes more and more pessimistic over time (higher types trade sooner than lower types) and asks for lower and lower prices. The famous Coase conjecture result is that without a deadline, as commitment disappears, the seller reduces its prices faster and faster and in the limit trade is efficient (no delay). We show that deadlines dramatically change the outcomes. A deadline provides the seller with a lower payoff bound that she can achieve by making unacceptable offers until the deadline. Our first main result (Proposition 2) is that as commitment disappears, the seller's equilibrium payoff converges to this lower bound: she obtains a payoff equal to the outside option of just waiting for the deadline. However, trade and prices do not converge to the standard Coase conjecture outcome (immediate trade and all types trading at one price) or the outside option (trade only at the deadline), but rather in the limit trade happens gradually over time. The price paid by each type is equal to the discounted price that this type would pay at the deadline if the seller adopted from now on the wait-till-deadline strategy. This property of prices is important to satisfy equilibrium conditions: if prices were higher, the seller would like to speed up trade; if they were lower, she would prefer to wait for the deadline. Finally, the speed at which equilibrium prices drop over time (which is one-to-one related to the speed at which the seller screens the types) assures that no buyer type wants to delay or speed up trade.

Once the deadline is reached and the last take it or leave it offer is made, there is a large probability that it is accepted, but there is also the possibility that the offer is not accepted. In that case, the players get their disagreement payoffs. Cramton and Tracy (1992) report that 57 percent of the labor negotiations in their sample end in disputes (strikes, 10 percent; holdouts, 47 percent; and lockouts, 0.4 percent). This may seem surprising since the failure to reach an agreement could be very inefficient. For example, when the existing contract between the NHL and its players expired on September 15, 2004 without having reached an agreement, the entire season was cancelled.

The tractable characterization of the continuous time limit allows us to perform comparative statics. We are able to obtain a series of testable predictions that resonate very well with some of the available evidence. For example, in Proposition 4 we show that the likelihood that an agreement will be reached is increasing in the loss of surplus from not doing so. This is consistent with the findings of Gunderson, Kervin, and Reid (1986) that strikes are more likely to occur (in Canada) when the efficiency losses from a strike are lower.⁷ This might also be an explanation for why strikes in professional sports leagues are very infrequent relative to other activities. Parties might purposefully commit themselves to a painful outcome in hope that this would induce them to reach an agreement. This was very apparent in the recent negotiations by a congressional committee that had to agree on \$1.2 trillion in budget savings (over ten years) by November 23, 2011. Failure to reach

⁶See Ausubel, Cramton, and Deneckere (2002) for a survey of the literature.

⁷Our model should be interpreted as capturing the labor negotiations before the end of the current contractual agreement or the preset strike date.

an agreement by the deadline was supposed to trigger automatic across-the-board budget cuts. Representative Chris Van Hollen, a democratic member of the committee, commented on these cuts by saying:

There's this sword of Damocles hanging over the process, meaning that if we're not able to reach agreement we're going to see across-the-board, indiscriminate cuts in both defense and nondefense so I think all those factors should focus all the minds of the members of the committee....

A second comparative static result is that bleaker prospects from disagreement lead to more of the agreements taking place on the “eleventh hour.” That is, although there is more agreement, most of it is concentrated at the deadline. That prediction, that lower disagreement payoffs not only increase the chances of compromise but also make the deadline effect more pronounced, while consistent with the anecdotes we discussed above, provides a good empirical test of the theory.

The last comparative static result we want to highlight is how the overall efficiency of the equilibrium varies with the parameters of the model. First, we show that for intermediate deadlines, the ex ante expected efficiency can be higher if passing the deadline is very costly to the parties—those swords of Damocles can play a positive role ex ante. Second, especially if the past-deadline inefficiency is small, the efficiency can be nonmonotone in the time to deadline: high for short and very long deadlines but smaller for intermediate ones. Finally, unlike in the standard Coase conjecture literature without deadlines, it is sometimes the case that if the seller had the commitment power to stick to its first offer until the deadline, the overall efficiency would be higher than in the noncommitment limit (it happens when the deadline is not too long).

Two other papers considered the continuous time limit of the ST model for the case in which the opportunity to trade disappears at the deadline. Güth and Ritzberger (1998) studied the case of a uniform distribution and have shown that in the noncommitment limit the seller's value converges to what she can attain from waiting for the last period.⁸ Ausubel and Deneckere (1992) considered the richer class of distributions that we study and observed that prices converge to the discounted monopolist price (see footnote 38 in their paper). Relatively to these, the main contribution of our paper is that we study the determinants of the deadline effect, a topic not addressed in these papers.

A closely related paper to ours is Hart (1989). That paper studies a model similar to ours to try to explain duration of strikes and labor disputes. In that model, there is also an exogenous deadline T after which the firm is facing a “crunch.” The main difference from our paper is the way the crunch is modeled: we assume that at T some of the value of the firm is lost—for example, because if the firm does not resolve the dispute by T , a major supplier is lost. In Hart (1989), at T the discounting increases. That leads to a major difference: in a model like Hart (1989) as the commitment to make offers disappears, disputes are again resolved immediately and efficiently; in a model with a discrete cost at T , inefficient delay persists even in the noncommitment limit.⁹

⁸They also show the Coase conjecture result in case the deadline goes to infinity and allow the traders to have different discount rates.

⁹To avoid the Coase conjecture, Hart (1989) explicitly focuses on the discrete time model.

Spier (1992) studies a model of pretrial negotiations and her section with exogenous deadlines is also relevant to our paper. The main difference from our model is that the social cost of delay is independent of type because legal costs are independent of the defendant's type. Our model applies to pretrial negotiations if the defendant's legal costs are proportional to his type. That would be the case, for example, if the plaintiff would restrict a use of an asset (a patent or a real estate) until the dispute is resolved. Similarly, the defendant may not be able to sell an asset or secure outside investment until the case is over (and the deadline may represent a loss of such outside opportunity).¹⁰ The difference in results is that in Spier (1992) offers are increasing over time while in our model they are decreasing (although in both models the distribution gets weaker over time). Also, we get a unique equilibrium in which each buyer type has a uniquely optimal time to trade. In her model, there are multiple equilibria and all defendant types that settle are completely indifferent over the time to settle. The similarity is that both models can deliver a deadline effect (however, in case of a binary distribution Spier (1992) obtains a U-shaped distribution of agreement times, while in our model there is no atom at $t = 0$).

Cramton and Tracy (1992) construct a more detailed model of wage bargaining. They seek to explain why in many instances when contracts are not renegotiated in time, unions choose to continue working under the old contract instead of starting a strike. This is referred to as a holdout. They assume a holdout is inefficient (for example, as a result of working to rule). Their model starts with the old contract already expired and the choice of the unions of what regime they want to be in while they continue negotiating. Our model should rather be interpreted as capturing the negotiation before the old contract expires. The ex post choice of threat by the union would still be relevant in our model since it would affect the disagreement payoffs which could be modeled as arising from the game described in Cramton and Tracy (1992). In this sense, we see our paper as complementary to theirs since it allows determining when the firm would reach a holdout.

In recent work, Hörner and Samuelson (2011) consider the case in which there are no costs of delaying trade until the deadline. Mapping a special case of their model to our environment, they show that with no discounting the seller makes unacceptable offers until the last moment and then makes the monopolist offer. In contrast, we show that with discounting, which makes delay costly, all offers are serious and trade takes place before the deadline with positive probability. With strict incentives for an early agreement, it is more surprising that a large fraction of the trades are delayed until the deadline. Moreover, the delay costs provide predictions about the relative probability of the agreement happening at the deadline or earlier.

On the more technical side, our paper is also related to bargaining models with interdependent valuations, as in Olsen (1992) and Deneckere and Liang (2006). The reason is that although the buyer value is independent of seller cost, by trading today the seller gives up the option of trading at the deadline. That opportunity cost is correlated with the value of the buyer. The main difference is that in our model the interdependence is created endogenously by the deadline and that the game is necessarily nonstationary.

¹⁰ Also, it may be that the higher the defendant's type the more it costs him to prevent the plaintiff from discovering damning evidence.

The next section presents the general model and a characterization of the unique equilibrium of the game. Section II characterizes the limit of the equilibria as offers can be revised continuously. In Section III, we then analyze how the deadline effect and division of surplus depend on the disagreement payoffs. Finally, Section IV studies two benchmark cases. First we look at the case when the opportunity to trade and hence all surplus disappears at T . Then we look at opposite case in which there is efficient trade after reaching the deadline (for example following the release of the private information). In that section, we also discuss how the overall efficiency of the equilibrium depends on the deadline and other parameters of the model.

I. The Model

There is a seller (a she) and a buyer (a he). The seller has an indivisible good (or asset) to sell. The buyer has a privately known type $v \in [0, 1]$ that represents his valuation of the asset. Types are distributed according to a c.d.f. $F(v) = v^a$.¹¹ We denote its density by $f(v)$. The seller's value of the asset is zero.¹²

There is a total amount of time $T < \infty$ for the parties to try to reach an agreement. The seller is able to commit to the current offer for a discrete period of length $\Delta > 0$.¹³ The timing within periods is as follows. In the beginning of the period the seller makes a price offer p . The buyer then decides whether to accept or reject this price. If he accepts, the game ends. If he rejects, the game moves to the next period.¹⁴ If time T is reached the game ends.¹⁵

Seller's (behavioral) strategy at time t , denoted $P(p^{t-1}, T - t; \Delta)$, is a mapping from the histories of rejected prices, p^{t-1} , and the remaining time, $(T - t)$, to the current period price offer, p_t . Buyer's type v (behavioral) strategy at time t , denoted $A_v(p^t, T - t; \Delta)$, is a mapping from the history of prices (rejected plus current) and the remaining time, to a choice whether to accept or reject the current offer.

The payoffs are as follows. If the game ends in disagreement the buyer gets a discounted payoff $e^{-rT} \beta v$ and the seller gets $e^{-rT} \alpha v$ where r is the common discount rate. We assume $\alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$. For example, the case $\alpha = \beta = 0$ represents that the opportunity to trade disappears after the deadline. This is the case analyzed by ST. In contrast, the case $\alpha = 1 - \beta$ could represent the revelation of information at time T , leading to bargaining with perfect information and no efficiency loss at T . Intermediate cases could for example capture partial (or probabilistic) loss of efficiency if the two parties do not reach an agreement before a trial and have to incur litigation costs. For example, if the buyer does not obtain the asset by T , he may lose with some probability the opportunity of using it. Also, the

¹¹ This assumption guarantees that truncated versions of the distribution between $[0, k]$ have the same shape as the original distribution. This gives some stationarity to the problem and helps simplify the analysis. In addition, it satisfies the decreasing marginal revenue condition for the seller's problem.

¹² The only nontrivial assumption about the range of v and the seller's value is that the seller's value is no lower than the lowest buyer's value—i.e., the “no-gap case.” The rest is a normalization.

¹³ For notational simplicity we only consider values of Δ such that $\frac{T}{\Delta} \in \mathbb{N}$.

¹⁴ Ausubel and Deneckere (1992) have shown that for small Δ , even if allowed to, the seller would not make any reasonable offer in equilibrium if the buyer gets to make the last offer.

¹⁵ The model can easily be extended to allow for the arrival of events before T , for example stochastic deadlines, as in Fuchs and Skrzypacz (2010).

model in Cramton and Tracy (1992) would deliver some intermediate values for a setting of wage negotiations.¹⁶

If the game ends with the buyer accepting price p at time t , then the seller's payoff is $e^{-rt} p$ and the buyer's payoff is $e^{-rt}(v - p)$.¹⁷

A. Equilibrium Definition

A complete strategy for the seller $\mathbf{P} = \{P(p^{t-1}, T - t; \Delta)\}_{t=0}^{t=T}$ determines the prices to be offered in every period after any possible price history.¹⁸ As usual (in dynamic bargaining games), in any equilibrium the buyer types remaining after any history are a truncated sample of the original distribution (even if the seller deviates from the equilibrium prices). This is due to the *skimming property* which states that in any equilibrium after any history of offered prices p^{t-1} and for any current offer p_t , there exists a cutoff type $\kappa(p_t, p^{t-1}, T - t; \Delta)$ such that buyers with valuations exceeding $\kappa(p_t, p^{t-1}, T - t; \Delta)$ accept the offer p_t and buyers with valuations less than $\kappa(p_t, p^{t-1}, T - t; \Delta)$ reject it. Best responses satisfy the skimming property because it is more costly for the high types to delay trade than it is for the low types. We can hence summarize the buyers' strategy by $\kappa = \{\kappa(p_t, p^{t-1}, T - t; \Delta)\}_{t=0}^{t=T}$.

DEFINITION 1: A pair of strategies (\mathbf{P}, κ) constitutes a *Perfect Bayesian equilibrium* of the game if they are mutual best responses after every history of the game. That requires:

- (i) Given the buyers' acceptance strategy, every period (after every history) the seller chooses her current offer as to maximize her current expected discounted continuation payoff.
- (ii) For every history, given the seller's future offers which would follow on the continuation equilibrium path induced by (\mathbf{P}, κ) , the buyers' acceptance choice of the current offer is optimal.

Being a finite horizon game, the equilibrium of the game can be solved by backward induction. As we show formally in the proof, with our distributional assumptions, given any period and any cutoff k induced by (\mathbf{P}, κ) and the history so far, the seller problem has a unique solution. Therefore, the continuation equilibrium is unique and depends on the history only via the induced cutoff and the remaining time, k and $(T - t)$. That allows us to simplify notation: let the current equilibrium price be denoted by $p = P(k, T - t; \Delta)$. Then the next period price is $P(\kappa(p, k, T - t; \Delta), T - (t + \Delta); \Delta)$, and so on.

¹⁶ Intermediate cases can also arise if we assume one of the parties will randomly get to make a last take it or leave it offer after T . (Assuming that if this last offer is not accepted the opportunity to trade is lost.)

¹⁷ We focus on the case $\Delta \rightarrow 0$, i.e., no commitment power, so it is more convenient to count time in absolute terms rather than in periods. Period n corresponds to real time $t = n\Delta$.

¹⁸ At $t = 0$ there is no history of prior prices, so $P(\emptyset, T; \Delta)$ should be simply interpreted as $P(T; \Delta)$. The same applies to the buyer's strategy.

The equilibrium (\mathbf{P}, κ) induces a decreasing step function $K(t, T - t; \Delta)$ which specifies the highest remaining type in equilibrium as a function of time passed and time remaining (with $K(0, T; \Delta) = 1$), and a decreasing step function $\Upsilon(v; \Delta)$ (with $\Upsilon(1; \Delta) = 0$) which specifies the time at which each type v trades (if it trades at all in equilibrium). For notational purposes, we let $k_+ = \kappa(P(k, T - t; \Delta), k, T - t; \Delta)$ denote the highest remaining type at the beginning of the next period given current cutoff k and the strategies (\mathbf{P}, κ) .

Let $V(k, T - t; \Delta)$ be the expected continuation payoff of the seller given a cutoff k with $(T - t)$ time left and the strategy pair (κ, \mathbf{P}) . For $t < T$ we can express $V(k, T - t; \Delta)$ recursively as

$$(1) \quad V(k, T - t; \Delta) = \left(\frac{F(k) - F(k_+)}{F(k)} \right) P(k, T - t; \Delta) + \frac{F(k_+)}{F(k)} e^{-\Delta r} V(k_+, T - (t + \Delta); \Delta),$$

and for $t = T$, we have

$$(2) \quad V(k, 0; \Delta) = \left(\frac{F(k) - F(k_+)}{F(k)} \right) P(k, 0; \Delta) + \frac{F(k_+)}{F(k)} \int_0^{k_+} \alpha v \frac{f(v)}{F(k_+)} dv.$$

For $t < T$, the seller's strategy is a best response to the buyer's strategy $\kappa(p, t; \Delta)$ if

$$(3) \quad P(k, T - t; \Delta) \in \arg \max_p \left(\left(\frac{F(k) - F(\kappa(p, k, T - t; \Delta))}{F(k)} \right) p + \frac{F(\kappa(p, k, T - t; \Delta))}{F(k)} e^{-\Delta r} V(\kappa(p, k, T - t; \Delta), T - (t + \Delta); \Delta) \right).$$

At $t = T$, the seller's strategy is a best response to the buyer's strategy $\kappa(p, T; \Delta)$ if

$$P(k, 0; \Delta) \in \arg \max_p \left(\frac{F(k) - F(\kappa(p, k, 0; \Delta))}{F(k)} p + \frac{F(\kappa(p, k, 0; \Delta))}{F(k)} \int_0^{\kappa(p, k, 0; \Delta)} \alpha v \frac{f(v)}{F(\kappa(p, k, 0; \Delta))} dv \right).$$

These best response problems capture the seller's lack of commitment: in every period she chooses the price to maximize her current value (instead of committing to a whole sequence of prices at time 0).

Necessary¹⁹ conditions for the buyer's strategy κ to be a best response is that given the expected path of prices the cutoffs satisfy:

¹⁹They turn out to be also sufficient.

For $t < T$:

$$(4) \quad \underbrace{k_+ - P(k, T - t; \Delta)}_{\text{trade now}} = e^{-\Delta r} \underbrace{(k_+ - P(k_+, T - (t + \Delta); \Delta))}_{\text{trade tomorrow}}.$$

For $t = T$:

$$(5) \quad \underbrace{k_+ - P(k, 0; \Delta)}_{\text{trade now}} = \underbrace{\beta k_+}_{\text{disagreement payoff}}.$$

Following the proof strategy of theorem 6 in ST we can establish the following result:

PROPOSITION 1: *The game has a unique Perfect Bayesian equilibrium. The equilibrium pricing function $P(p^{t-1}, T - t; \Delta)$ depends only on the history of past prices via the cutoff type k that they induce and it is linear in k . The seller's value $V(k, T - t; \Delta)$ is also linear in k :*

$$P(p^{t-1}, T - t; \Delta) = P(k, T - t; \Delta) = \gamma_t k$$

$$V(k, T - t; \Delta) = \frac{a}{a + 1} \gamma_t k,$$

where

$$\gamma_T = (1 - \beta) \left(1 + a \frac{1 - \alpha - \beta}{1 - \beta} \right)^{-\frac{1}{a}}$$

and for $t < T$,

$$(6) \quad \gamma_t = (1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}) \left(\frac{1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}}{(a + 1)(1 - e^{-r\Delta}) + e^{-r\Delta} \gamma_{t+\Delta}} \right)^{\frac{1}{a}}.$$

The proof is by induction and is delegated to the Mathematical Appendix. The buyer strategy (for on and off-path offers) is described in the last paragraph of the proof. If the seller makes an off-equilibrium path offer, the inductive proof shows that the continuation strategy is unique and depends only on the cutoff type that accepts that off-path offer.

II. Limit of Equilibria as $\Delta \rightarrow 0$.

We now take the limit of the equilibria described in the previous section as $\Delta \rightarrow 0$. We show that the limit expressions are much simpler. While much of the previous literature refers to Δ as the “bargaining friction” we prefer to refer to it as measure of the seller’s ability to commit to the current offer.

Note that Δ affects equilibrium strategies only via its effect on γ_t . The difference equation for γ_t , (6), converges to a simple differential equation as $\Delta \rightarrow 0$, with a boundary condition γ_T which does not depend on Δ . The solution of that differential equation is $\gamma_t = e^{-r(T-t)} \gamma_T$. That allows us to obtain the following characterization of the continuous time limit of equilibria:²⁰

PROPOSITION 2: As $\Delta \rightarrow 0$:

- (i) $V(k, T - t; \Delta) \rightarrow V(k, T - t) = e^{-r(T-t)} V(k, 0)$
- (ii) $P(k, T - t; \Delta) \rightarrow P(k, T - t) = e^{-r(T-t)} P(k, 0)$
- (iii) $K(t, T - t; \Delta) \rightarrow K(t, T - t) = \exp\left(\frac{1}{\gamma_T} e^{rT} (e^{-rt} - 1)\right)$

and an atom $m(T)$ of trade at time $t = T$:

$$\begin{aligned} m(T) &= K(T, 0)^a - (\kappa(P(K(T, 0), 0), K(T, 0), 0; 0))^a \\ &= \exp\left(\frac{a}{\gamma_T} (1 - e^{rT})\right) \left(1 - \left(\frac{\gamma_T}{1 - \beta}\right)^a\right), \end{aligned}$$

where $P(k, 0)$ and $V(k, 0)$ are optimal price and payoff at T (independent of Δ).

COROLLARY 1 (Atom at the Deadline): If $\alpha + \beta < 1$ then trade is continuous for $t \in [0, T)$ but there is a positive measure of trade at time T .

Beyond being more tractable, the limiting expressions have clear economic interpretation. Consider first the value for the seller. $V(k, T - t)$ is simply what she would get from just waiting and making the last offer.²¹ This implies that the seller's value is driven down to her outside option of simply waiting to make her last offer. The uninformed party cannot capture more than her reservation value once her ability to commit to an offer disappears.²²

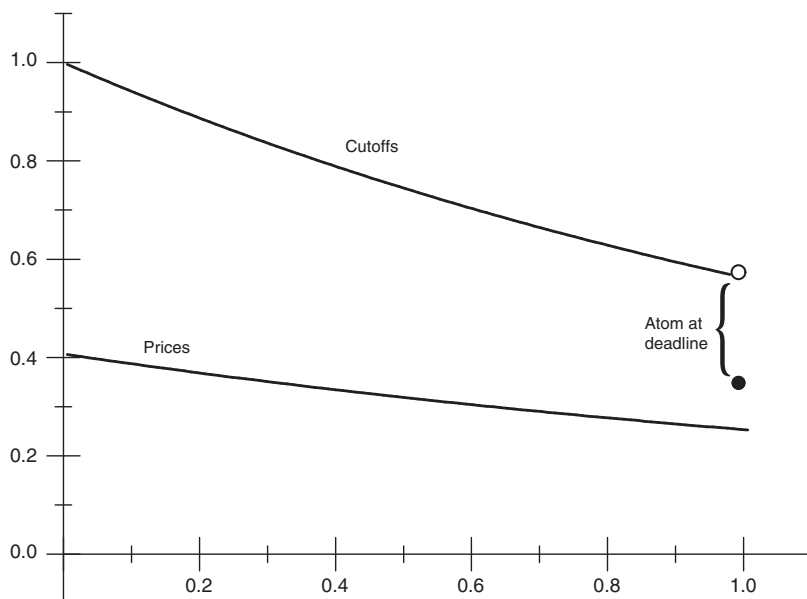
Second, the equilibrium prices display a no-regret property: the seller is indifferent between collecting $P(k, T - t)$ from type k today or getting what this type would contribute to her value upon reaching the deadline, $e^{-r(T-t)} P(k, 0)$. The equation (iii) describing $K(t, T - t)$ is a bit more complicated but it is a solution to an intuitive differential equation. It follows from taking the continuous time limit of the buyers best response condition given in (4):

$$(7) \quad r(k - P(k, T - t)) = -P_k(k, T - t) \dot{K} - P_t(k, T - t)$$

²⁰ As we discussed in the introduction, for the case $\alpha = \beta = 0$ (ii) appears in Ausubel and Deneckere (1992) and when in addition $a = 1$ Güth and Ritzberger (1998) obtain (i).

²¹ Even though this is not what she actually does in equilibrium.

²² This is similar to the result in Fuchs and Skrzypacz (2010), which is a stationary bargaining problem with outside options that arrive stochastically over time.

FIGURE 1. EQUILIBRIUM PATH OF p AND K ; NOTE THE ATOM AT $T = 1$

where P_k and P_t are the derivatives of $P(k, T - t)$ (from (ii)) with respect to k and t , respectively. The RHS represents the change in price that results from the horizon getting closer and the seller updating her beliefs downwards. The LHS captures the costs of delay for the buyer in terms of interest lost on the profit. Solving (7) with a boundary condition $K(0, T) = 0$ yields (iii).

In Figure 1, we illustrate the equilibrium path of cutoff types and prices over time.²³ As described in Proposition 2, trade takes place smoothly over time except for the last instant before the deadline; at this point there is an atom of trade $m(T)$. In the plotted example the “deadline effect” includes approximately types between 0.6 and 0.4. Despite this last rush of agreements, some types do not reach an agreement—in the example these are types below 0.4. In general the probability of agreement is given by $\Pr(\text{agreement}) = 1 - K_{T+}^a$, where K_{T+} is the cutoff after the last equilibrium offer.

Time to the Deadline T .—Next, we look at how the time to the deadline affects the seller’s payoff and the terms of trade. We also consider the limit as we push the deadline towards infinity to allow for the model to approach the no deadline models of FLT and GSW.

PROPOSITION 3: *For every $k > 0$ and $t \leq T$, $V(k, T - t)$ and $P(k, T - t)$ are decreasing in rT and go to 0 uniformly as $rT \rightarrow \infty$. The probability of agreement is increasing in rT and goes to 1 as $rT \rightarrow \infty$. $K(t, T - t)$ and the last atom $m(T)$ are decreasing in rT and go to 0 as $rT \rightarrow \infty$.*

²³The parameters used are $T = 1$, $r = 10$ percent, $a = 1$, $\alpha = \beta = \frac{1}{4}$.

PROOF:

This proposition follows from the characterization of the equilibrium objects and the fact that γ_t is decreasing in rT and for any finite t (and in particular for $t = 0$) $\lim_{rT \rightarrow \infty} \gamma_t = 0$.

COROLLARY 2:

- (i) (*Delay*): For all $0 < rT < \infty$ the expected time to trade is strictly positive.
- (ii) (*Coase conjecture*): As $rT \rightarrow \infty$, the expected time to trade and transaction prices converge to 0 for all types (i.e., $\Upsilon(v) \rightarrow 0$ pointwise for all $v > 0$ and $P(k, T - t) \rightarrow 0$ pointwise for all $k > 0$ and t).

Part (i) follows directly from our characterization, but the intuition is as follows: for there to be no delay in equilibrium the transaction prices for all types have to be close to zero, implying a seller's payoff close to zero, in particular, less than $e^{-rT} V(k, 0) > 0$. But that leads to a contradiction since the seller can guarantee himself that by just waiting for the last period. This indirect proof establishes that trade is necessarily inefficient for any equilibrium not only for our family of distributions, but for any distribution without a gap.²⁴

Part (ii) shows that our limit of equilibria converges to the equilibria in GSW and FLT: as we make the horizon very long (convergence of the model) trade takes place immediately and the buyer captures the entire surplus (convergence of equilibrium outcome). Note that ST had established that the same holds when one reverses the order of limits, first taking the limit as $T \rightarrow \infty$ (the FLT and GSW models) and then $\Delta \rightarrow 0$. Actually, like a lot of the previous literature, ST use a per period discount rate $\delta = e^{-r\Delta}$ and take the limit as $\delta \rightarrow 1$. In the finite horizon case, it actually matters if $\delta \rightarrow 1$ as a result of $\Delta \rightarrow 0$ or $r \rightarrow 0$. The latter would make the whole game essentially a static game since $rT \rightarrow 0$.

When Do Deadlines Matter?—The previous proposition established that for large rT the equilibrium outcome converges to the Coasian outcome in a game without deadlines. To illustrate for which deadlines the equilibrium is far from that deadline, in Figure 2 we plot the seller's value at time zero (dashed) and the initial price demanded (solid) for the uniform ($a = 1$) case for $\alpha = \beta = 0.25$. As we can see from the plot, as the (normalized) horizon until the deadline extends we converge to the Coasian results, initial prices and seller's value are very close to 0, for $rT > 1$, but substantially away from the Coase conjecture for $rT < 0.2$.

Figure 3 graphs the probability of agreement (solid), atom size (dots), and percentage of trades that take place at the deadline (dashed). The plot illustrates that, although for short horizons most of the trade takes place in the last offer, as the horizon increases this changes. Furthermore, when rT is greater than one, trade takes place with very high probability and it mostly takes place before the deadline.

²⁴ By the same argument, even if there is a gap between the lowest buyer type, \underline{v} , and the seller cost (normalized to zero), trade must be inefficient in equilibrium even as $\Delta \rightarrow 0$ if $\underline{v} < e^{-rT} V(\bar{v}, 0; \Delta)$, where \bar{v} is the highest type.

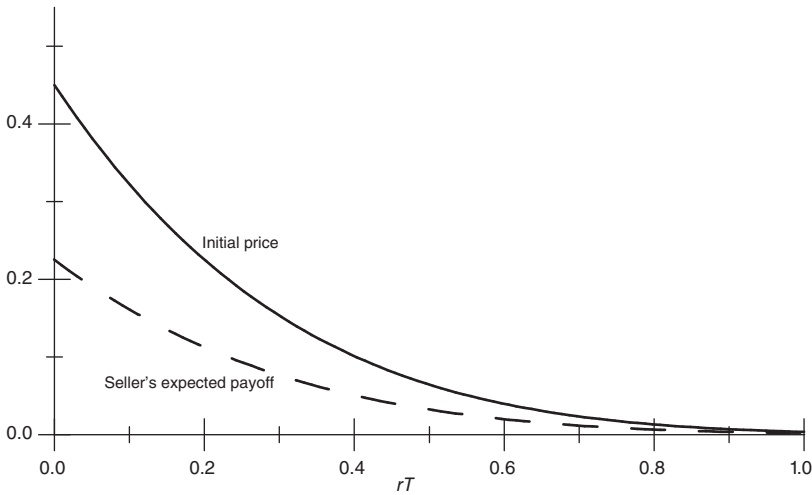


FIGURE 2. INITIAL PRICE AND SELLER'S EXPECTED PAYOFF

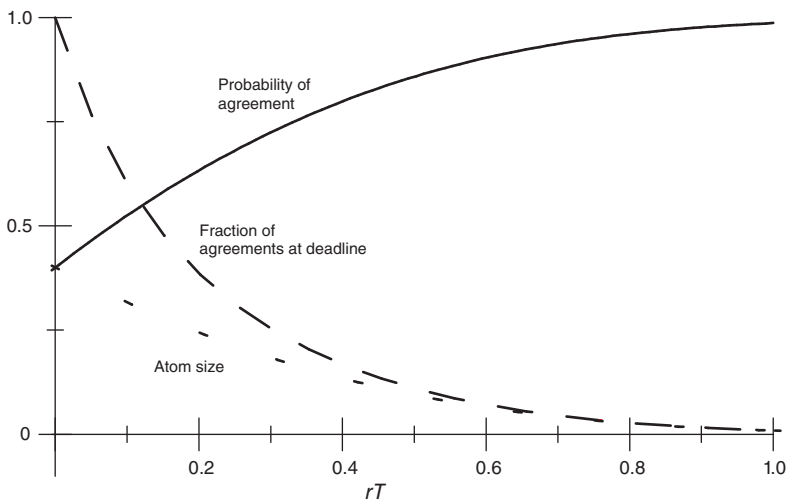


FIGURE 3. PROBABILITY OF TRADE AND THE DEADLINE EFFECT AS THE FUNCTION OF THE HORIZON

In summary, in these examples, if the discount rate is between 10 percent and 20 percent, the deadlines would have to be over 1–2 years not to matter. For many of the practical applications of bargaining with deadlines we described in the Introduction, the Coasian limit may not be a useful benchmark.

Deterministic versus Stochastic Deadlines.—It is worth noting that it is quite different if we face a fixed deadline date rather than a stochastic deadline that arrives at a Poisson rate λ after which the opportunity to trade disappears, even if from time zero perspective the expected time available to reach an agreement is the same ($\frac{1}{\lambda} = T$). This difference arises because knowing that the game ends at T allows the seller to make a credible last take it or leave it offer at T . This possibility allows him

to extract a positive amount of surplus. Instead, a stochastic loss of the opportunity of trade that arrives as a surprise is equivalent to having a higher discount rate $\hat{r} = r + \lambda$ and would lead to immediate trade with the buyer capturing the entire surplus. This is a difference between our model and that discussed in Hart (1989).

If we instead allowed the seller to make a last take it or leave it offer when the stochastic deadline materialized, the outcome would be much closer (but not the same) to what we would obtain with a deterministic deadline. In fact, the seller would prefer the stochastic deadline. In this case, her value would be $\frac{\lambda}{\lambda + r} V(1, 0)$ instead of $e^{-\frac{r}{\lambda}} V(1, 0)$ and $\frac{\lambda}{\lambda + r} > e^{-\frac{r}{\lambda}}$. This follows simply because the present value of a dollar is a convex function of the time at which it is generated. Therefore, a mean preserving spread increases value.

A. Heuristic Derivation of the Limit

Proof of Proposition 2 uses the explicit closed form construction of equilibria in discrete time. To obtain this strong characterization, we limited our analysis to a family of distributions that has the nice property that any truncation has the same shape as the original distribution.²⁵ We conjecture that the result is more general, that for any well-behaved $f(v)$, taking a limit of any sequence of PBE of the discrete time games as $\Delta \rightarrow 0$, we would get $V(k, T - t) = e^{-r(T-t)} V(k, 0)$, $P(k, T - t) = e^{-r(T-t)} P(k, 0)$ and $K(t, T - t)$ is a solution to (7) with a boundary condition $K(0, T) = 1$. This conjecture is likely to be true if in the limit the equilibria become Markovian (the continuation payoffs depend on the history only via k and $T - t$) and in the limit there are no atoms of trade before T . In that case, heuristically the seller's best response problem would be to choose the speed with which to skim buyer types, \dot{K} and the value of that strategy would be

$$(8) \quad rV(k, T - t) = (P(k, T - t) - V(k, T - t)) \frac{f(k)}{F(k)} (-\dot{K}) \\ + V_k(k, T - t) \dot{K} + V_t(k, T - t).$$

If an interior \dot{K} is optimal in the limit²⁶, then because the RHS is linear in \dot{K} , it must be that the coefficients on \dot{K} add up to zero:

$$(9) \quad (P(k, T - t) - V(k, T - t)) \frac{f(k)}{F(k)} = V_k(k, T - t).$$

If (9) is satisfied, then all terms with \dot{K} drop out of (8) and we get

$$rV(k, T - t) = V_t(k, T - t).$$

²⁵ Our distributional assumption also makes the seller's problem quadratic and hence easier to solve in closed form.

²⁶ If the limit $K(t, T - t)$ is continuous but not differentiable at t , then the reasoning applies to the right derivative.

Together with the boundary condition $V(k, 0)$, this differential equation has a unique solution $V(k, T - t) = e^{-r(T-t)} V(k, 0)$; the seller value at any moment of the game and after any history is equal to the value of the outside option of waiting for the deadline. Finally, plugging it into (11) we get $P(k, T - t) = e^{-r(T-t)} P(k, 0)$ as claimed (note that by the envelope theorem, $\frac{\partial F(k) V(k, 0)}{\partial k} = P(k, 0) f(k)$). The economic intuition behind this heuristic reasoning is that prices higher than $e^{-r(T-t)} P(k, 0)$ would make the seller want to speed up trade, while prices lower than that would make him want to stop it and wait. The Coase conjecture forces manifest themselves in the linearity of the limit objective function (8) in \dot{K} .

Generalizing the Deadline Effect.—Conditional on reaching the deadline, as long as there are strict gains from trade (i.e., passing the deadline has a discrete cost), there will be a mass of trade at the deadline. This holds for general distributions, $F(v)$, and more general disagreement payoffs than we study. This claim can be proven using static analysis.

In what situations would we reach the deadline? For a general $F(v)$ with support in $[0, 1]$, the deadline would be reached because the seller would never want to set a price of 0. In the “gap” case, i.e., a distribution of values with support in $[g, g + 1]$ for $g > 0$, it is possible that in equilibrium the deadline would not be reached at all, because the seller would be better off by trading at a price of g with all remaining types than delaying trade any further. If g is not too high, for small enough T the deadline would be reached and then we conjecture that the equilibrium path would be the same as in our limit (although the off-equilibrium path would be different than in our heuristic derivation).

As discussed at the beginning of this subsection, we conjecture that in the no-gap case with costly deadline-missing, it should hold for any $F(v)$ that in the (limit) equilibrium there is smooth trade before T and an atom of trade at the eleventh hour. While we find it intuitive, it turns out that proving this conjecture is much harder for general distributions. While it is easy to prove that the probability to trade n periods before deadline T converges pointwise for every n to zero as $\Delta \rightarrow 0$ (since otherwise the price would decrease discontinuously at T), this is not sufficient to prove that there would be no atoms of trade in the limit. First, as $\Delta \rightarrow 0$, the number of periods in the game grows to infinity and hence pointwise convergence is not sufficient. Second, even if we could prove that the probability of trade goes to zero uniformly for all $k \in \{1, \dots, \frac{T}{\Delta}\}$, it would not be enough since if that probability goes to zero at a rate slower than Δ over a range of periods, the limit $K(t, T - t)$ could be discontinuous.²⁷ One of the difficulties is that for a general $F(v)$ the seller’s maximization problem may have multiple solutions in some periods. That makes it hard to prove uniqueness of the equilibrium limit and its properties and in particular establishing rates of convergence (and would require more complex methods than our current analysis).

²⁷ Similarly, if the probability of trade in a given period goes to zero over at a rate faster than Δ over a range of times, then in the limit $K(t, T - t)$ could be constant over a range of time, unlike in our equilibria.

III. Outside Options and the Deadline Effect

In this section, we analyze how the deadline effect and division of surplus depend on the disagreement payoffs α and β . For this analysis it is useful to also define a *conditional atom*, $\mu(T) = m(T)/K^a(T, 0)$, i.e., the probability of trade at the deadline conditional on reaching it (as opposed to $m(T)$ which is from the ex ante perspective). Given our distributional assumption, it has a very simple form:

$$(10) \quad \mu(T) = 1 - \left(a \frac{1 - \alpha - \beta}{1 - \beta} + 1 \right)^{-1}.$$

We start with the following general results about the probability of trade and the deadline effect:

PROPOSITION 4:

- (i) *The fraction of negotiations that reach agreement conditional on reaching the deadline (the conditional atom) is decreasing in α and β . If $\alpha + \beta$ is held fixed, it is also decreasing in α/β .*
- (ii) *The unconditional probability that agreement is reached at the deadline ($m(T)$) is decreasing in β , it is also decreasing in α for small rT . If $\alpha + \beta$ is held fixed, it is also decreasing in α/β .*
- (iii) *The overall probability of agreement is decreasing in β , it is also decreasing in α for small rT . If $\alpha + \beta$ is held fixed, it is also decreasing in α/β .*
- (iv) *If rT is sufficiently small, the fraction of all agreements that take place at the deadline is decreasing in α and β . If $\alpha + \beta$ is fixed, the fraction is increasing in α/β .*

The proof is by direct manipulation of the expressions we obtained above.

In words, the proposition states that the bleaker the prospects if they do not reach an agreement (a lower α or β) the more likely they will reach one, as one would expect from a static model. More interestingly, the bleaker are the prospects, proportionally more of the trade takes place in the last instant when it is clear that not agreeing would be bad for both. At the same time, if we keep the inefficiency of disagreement fixed, the stronger the seller's position, there are fewer agreements (intuition: the seller suffers from less adverse selection if he waits past deadline and if α is higher, waiting is less costly) and they are more likely to happen right at the deadline.

Studying the incidence of strikes from Canadian data Gunderson, Kervin, and Reid (1986) find that strikes are more likely to occur (passing T) when the cost of not reaching an agreement by the strike date are lower (higher α and β).²⁸

²⁸ Also in line with our results, anecdotal evidence suggests that pretrial settlements are more likely when the legal system is more inefficient.

To assess how small is “small enough” in the proposition we have performed several numerical computations. For example in the uniform distribution case, $a = 1$, if we let $\alpha = \beta \leq \frac{1}{2}$ then as prospects get bleaker a larger fraction of agreements happen at the deadline even for $rT = 1$. In Figure 4, we graph the probability of agreement (solid) and atom size (dotted) and the percentage of agreements at T (dashed) for $a = 1, \alpha = \beta, r = 10$ percent and $T = 1$.

Beyond the probability of trade, one may be also interested in how α and β affect equilibrium prices and payoffs:

PROPOSITION 5:

- (i) *For every t , $V(k, T - t)$, $P(k, T - t)$ and $K(t, T - t)$ increase in α and decrease in β . If $(\alpha + \beta)$ is kept fixed they increase in α/β .*
- (ii) *The equilibrium price in each moment t increases in α and decreases in β .*

PROOF:

Part (i) is obtained by direct inspection of the limit equilibrium and the monotonicity of γ_T .

Part (ii) is derived by combining part (i) and that prices increase in k .

Summarizing, as the seller's disagreement situation improves, she obtains higher payoff in equilibrium and sets higher prices to all types and over time. The opposite is true if the buyer's disagreement payoff improves.

One way to relate our results to the data for the firms (buyers) bargaining with the unions (sellers) is to think of α/β as a function of the unemployment rate. When unemployment is higher the firm could have an easier time replacing workers and workers a harder time finding new employment so that β is higher and α is lower. Thus, we would expect that when bargaining their yearly contracts unions would settle for worse terms and there would a lower likelihood of not reaching an agreement (going on strike) when the unemployment rate is higher. Capturing this effect in the data is not simple since the unemployment rate is not independent of the distribution of firm profitability, captured by a .²⁹

IV. Two Benchmark Cases

We now discuss two benchmark cases.

Information Revealed at T .—Suppose that at time T the private information is revealed and hence there can be efficient trade i.e., $\alpha + \beta = 1$. In this case the equilibrium has some interesting properties. First, there is no longer an atom of trade in the last instant before the deadline. This is simply a manifestation of the no trade theorem. The buyer will not be willing to accept any price lower than $(1 - \beta)k$; on

²⁹Times of high unemployment might correspond to low values of a . See Section IV for comparative statics analysis w.r.t. a .

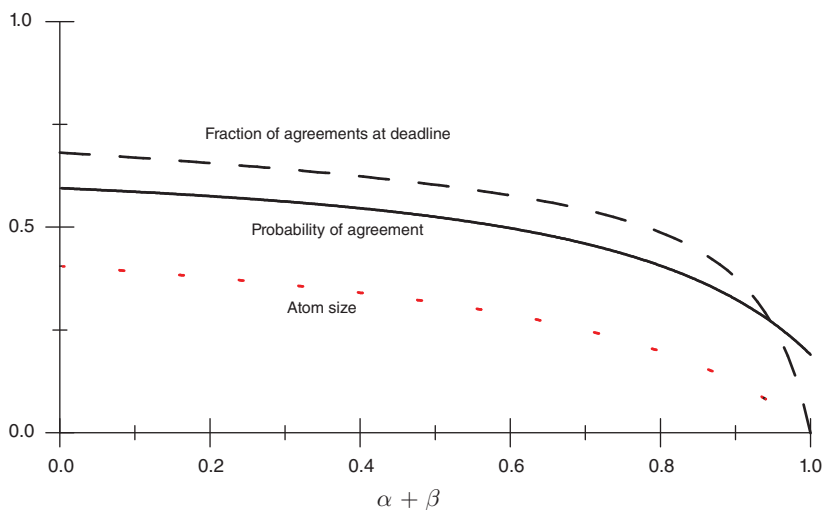


FIGURE 4. PROBABILITY OF TRADE AND RELATIVE DEADLINE EFFECT AS A FUNCTION OF DISAGREEMENT PAYOFFS

the other hand the seller would not want to offer any price lower than this since the next instant she expects to get that amount. Before the last instant, there will be trade since there is a cost of waiting until the deadline and hence the highest type buyer and seller can find mutually beneficial terms of trade. Second, when $\alpha + \beta = 1$, γ_T is independent of the distribution of types (to verify, see that γ_T in Proposition 1 becomes independent of a). As a result, the equilibrium price and acceptance strategies are also independent of the distribution of types. We obtain the following comparative statics in this benchmark case:

PROPOSITION 6: *Suppose $\alpha + \beta = 1$. Then, $V(k, T - t)$, $K(t, T - t)$ and $P(k, T - t)$ are increasing in α and for every $t \geq 0$. The probability trade takes place before T is decreasing in α .*

PROOF:

This proposition follows from noting that when $\alpha + \beta = 1$ we have that $\gamma_T = \alpha$. Using this fact, some simple computations lead to the results.

COROLLARY 3 (Coasian Extreme): *If $\alpha = 0$ and $\beta = 1$, then for all rT trade is immediate at a price of 0 and the buyer captures the entire surplus.*

Note that the key to this result is not the fact that $\alpha = 0$, but rather that $\beta = 1$. Even if the seller cannot capture anything upon disagreement, the fact that she can make a last take it or leave it offer at T gives her a lot of power. Yet, this power is proportional to $(1 - \beta)$. When $\beta = 1$, she effectively has no power since the buyer can capture the entire surplus by rejecting the last offer.

In the bargaining literature, the correlation of values can be important in determining the equilibrium outcome—see, for example, Deneckere and Liang (2006). In our model, setting $\alpha = 0$ represents independent values (the disagreement payoff

of the seller is independent of v). Still, we do not get the Coase conjecture result of immediate trade unless $\beta = 1$, because the seller can make a last take-it-or-leave-it offer at T and the expected payoff of that offer endogenously depends on v even if $\alpha = 0$. Only if $\beta = 1$ there is indeed no interdependence because the seller cannot extract anything from the last offer. It is in this case that we get immediate trade at a price of zero as the limit equilibrium outcome for every rT .

While so far we have focused on the time of agreement and the seller payoffs, in this case we can also say something about the buyer payoff and overall surplus:

COROLLARY 4: *Suppose $\alpha + \beta = 1$. For every $rT > 0$, the ex ante expected efficiency from trade is decreasing in α , and all buyer types are worse off as α increases.*

The loss of efficiency follows because $K(t, T - t)$ increasing in α implies that trade is slower for larger α (and since there is no atom of trade at T , more types trade after the information is revealed). The inefficiency arises from the discounting cost of delayed trade. In addition to trading later, buyers also suffer from paying higher prices.

Finally, we can say how the distribution parameter a affects prices and payoffs:

PROPOSITION 7: *Suppose $\alpha + \beta = 1$. Then, $V(k, T - t)$ and expected transaction prices are increasing in a . Expected time to trade is decreasing in a .*

PROOF:

Since the equilibrium price and acceptance strategies are independent of the distribution of types, the only effect of a higher a is that there are relatively more high types and since trade happens sooner and at higher prices with these types, the result follows.

Note that for a fixed buyer type v , differences in a would have no impact on its outcome. This is only true in the extreme with no efficiency loss upon reaching T . More generally, strategies do depend on a , a fixed buyer type would then face different outcomes when a is high than when it is low.³⁰

Lost Opportunity of Trade at T .—Now consider the benchmark case $\alpha = \beta = 0$. That is, if players do not reach an agreement the potential surplus from doing business is lost forever. In this case:

$$\gamma_T = \left(\frac{1}{1 + a} \right)^{\frac{1}{a}},$$

which is increasing in a . That implies the following comparative statics results:

PROPOSITION 8: *Suppose $\alpha = \beta = 0$. Then, $V(k, T - t)$, $K(t, T - t)$ and $P(k, T - t)$ are increasing in a .*

³⁰ See Proposition 8, which looks at the effect of changes in the distribution for the case $\alpha = \beta = 0$.

Note that this implies for example that a firm of a fixed type (profitability) v is less likely to reach an agreement with the unions when the overall economy is stronger (higher a) than when the economy is in a recession. As stated by Kennan and Wilson (1989), although hard to obtain in a model, this is consistent with some of the empirical studies on strikes:

Other aspects of the incidence and duration of strikes pose particularly difficult challenges. There is a well-established body of evidence (summarized in Kennan 1986), showing that there are more strikes in good economic times than in bad times, and Vroman (1989) and Gunderson, Kervin, and Reid (1986) have recently sharpened this result for US and Canadian data respectively, showing that the incidence of contract strikes is also procyclical.

Admittedly, comparative statics with respect to a are only one of the ways to capture changes in the environment, but it points out that the model is capable of matching complicated patterns in the data. To take it seriously to the data we think it would be necessary to enrich the model with the institutional details of a particular market, which is beyond the scope of this paper.

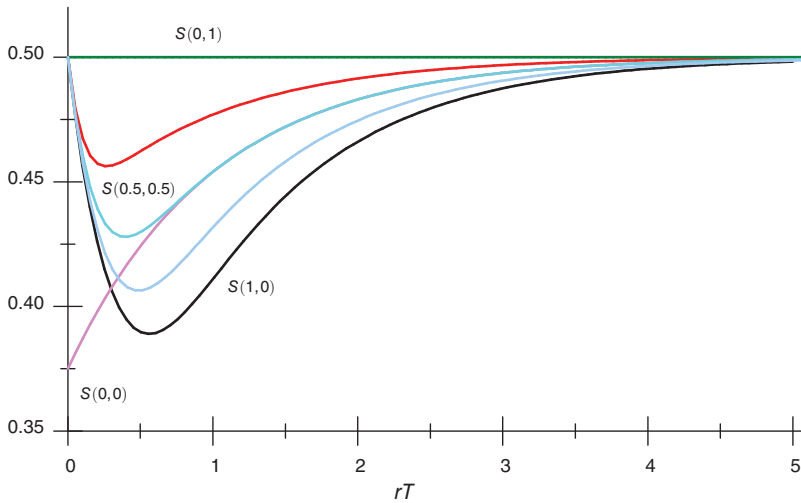
*Efficiency Comparisons.*³¹—One might wonder if the threat of an unattractive outcome upon reaching the deadline, for example automatic budget cuts as in the recent budget negotiations, would lead to more efficient or inefficient outcomes. In Figure 5 we plot the expected total surplus $S(\alpha, \beta)$ for various combinations of α and β as a function of how long the deadline is, rT . All computations are for $a = 1$ (buyer types distributed uniformly).

The first-best surplus in this case is $\frac{1}{2}$ (immediate trade for all types) and this value is attained with $S(0, 1)$ for any trading horizon rT (as stated in Corollary 3).

In the benchmark case of no inefficiency at the deadline, $\alpha + \beta = 1$, as we increase α , the total surplus decreases for any horizon, as stated in Corollary 4. Perhaps somewhat surprisingly, the effect of the deadline is nonmonotonic in this case. The intuition is that for very short horizons the deadline is reached very frequently but full efficiency is attained at that point. On the other extreme, when the horizon is very long, the standard Coase conjecture result re-emerges as established in Corollary 2 and trade takes place almost immediately and efficiency loss is also negligible. For intermediate horizon lengths there equilibrium has nontrivial inefficiency because there is a lot of delay.

The length of the horizon plays a very different role when $\alpha = \beta = 0$. In this case, for short horizons the seller is effectively a one-shot monopolist who would offer a price $\frac{1}{2}$, capture expected surplus $\frac{1}{4}$ with a corresponding buyer surplus equal to $\frac{1}{8}$ —the curve for $S(0, 0)$ starts at this surplus of $\frac{3}{8}$. Hence, for short horizons, the overall efficiency is higher if passing the deadline does not create discrete loss of surplus, $\alpha + \beta = 1$. For longer bargaining horizons and if the seller's share upon

³¹ We thank an anonymous referee for suggesting material in this section.

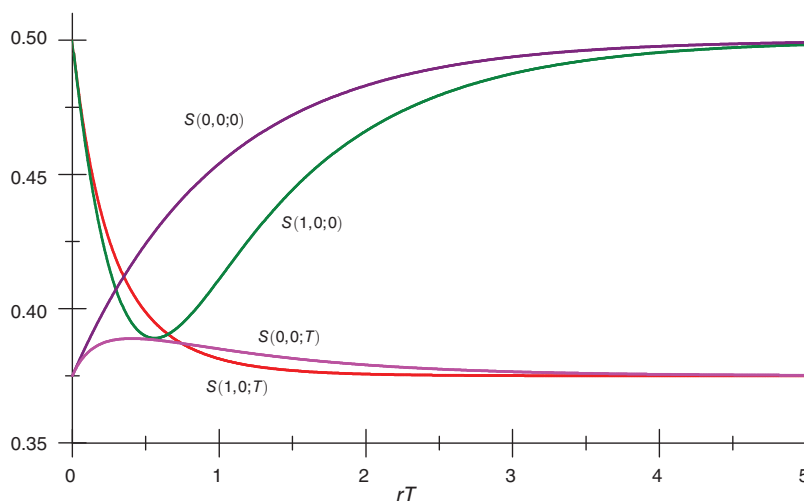
FIGURE 5. EXPECTED SURPLUS AS $\Delta \rightarrow 0$, $S(\alpha, \beta)$

reaching the deadline is greater than $\frac{1}{2}$ we can get the opposite ranking: as seen in this graph, for some rT , $S(0, 0) > S(\alpha, 1 - \alpha)$.

Another natural question regarding overall efficiency of the bargaining equilibrium is how it is affected by the frequency of offers, as measured by Δ . It turns out that the answer is not straightforward because it depends on the surplus upon reaching the deadline (α, β) , the horizon length (rT) and the distribution of types.³² Complete analysis is beyond the scope of this paper, but we illustrate possible answers by comparing the extreme case of $\Delta = T$ with $\Delta \rightarrow 0$ when values are uniformly distributed. In Figure 6 we show the expected equilibrium surplus indexed by α and β as well as by the frequency of offers, $S(\alpha, \beta, \Delta)$. The curves for $S(0, 0, 0)$ and $S(1, 0, 0)$ are the same as in the Figure 5, and the new curves are for $S(0, 0, T)$ and $S(1, 0, T)$.

For long horizons (rT large) the driving force determining the surplus is the frequency of offers. Frequent offers yield approximately efficient surplus $\frac{1}{2}$ and infrequent offers yield approximately the one-shot monopolist outcome $\frac{3}{8}$. When the deadline is very short, and hence very likely to be reached, the overall efficiency is driven by the loss of surplus from not reaching an agreement ($\alpha + \beta = 1$ versus $= 0$). Note that for $\alpha = 1$ the ranking of surplus for $\Delta \rightarrow 0$ and $\Delta = T$ depends on T and numerical calculations show that the same is true for many other levels of α and β . Hence, the Δ that maximizes overall efficiency depends on the length of the horizon and other parameters of the model. As a result, sometimes the need for the labor union representatives to get time-consuming response from the union members may improve not only their own payoff but overall efficiency as well.

³²For similar questions on how frequency of offers affects surplus, recall that the Coase conjecture literature with infinite horizon points out that small Δ are better for efficiency. Similar questions have been asked in Chen (2012) who studies name your own price auctions and Fuchs and Skrzypacz (2013) who study dynamic markets for lemons.

FIGURE 6. EXPECTED SURPLUS $S(\alpha, \beta, \Delta)$

V. Conclusions

With a parsimonious model that builds on the previous literature, we have captured the effects of deadlines on bargaining environments with one-sided asymmetric information. The most salient of the equilibrium features is the mass of agreements that take place at the “eleventh hour.” This is very much in line with the existing empirical and experimental data.

Our model predicts how postdeadline payoffs affect the division of surplus and the timing of agreements. The possibility to characterize the limit of equilibria in closed form, and to perform additional comparative statics analysis, opens the door to revisit some of the experimental data and suggests interesting avenues for future empirical tests.

MATHEMATICAL APPENDIX

PROOF OF PROPOSITION 1:

The proof is by induction and is similar to the one in ST. From equation (5), at time T a buyer with type k accepts prices lower than $(1 - \beta)k$.

$$\max_p p \left(\frac{k^a - \left(\frac{p}{1 - \beta} \right)^a}{k^a} \right) + \frac{1}{k^a} \frac{a}{a + 1} \alpha \left(\frac{p}{1 - \beta} \right)^{a+1},$$

which has a unique solution at

$$P(k, 0; \Delta) = k(1 - \beta) \underbrace{\left(1 + a \frac{1 - \alpha - \beta}{1 - \beta} \right)^{-\frac{1}{a}}}_{\equiv \gamma_T}.$$

The seller's payoff then is

$$V(k, 0; \Delta) = \gamma_T k \frac{a}{a+1}.$$

For a general t , the seller problem is

$$\begin{aligned} V(k, T-t; \Delta) = \max_p & p \left(\frac{k^a - \kappa(p, T-t; \Delta)^a}{k^a} \right) \\ & + e^{-r\Delta} V(k, T-(t+\Delta); \Delta) \frac{\kappa(p, T-t; \Delta)^a}{k^a}, \end{aligned}$$

where

$$(A1) \quad \kappa(p, T-t; \Delta) - p = e^{-r\Delta} (\kappa(p, T-t; \Delta) - P(\kappa, T-(t+\Delta); \Delta))$$

is the buyer's best response (necessary) condition.

In order to complete the proof by induction, assume that for all k_+ :

$$V(k_+, T-(t+\Delta); \Delta) = \frac{a}{a+1} \gamma_{t+\Delta} k_+ \quad \text{and} \quad P(k_+, T-(t+\Delta); \Delta) = \gamma_{t+\Delta} k_+.$$

Substituting the buyer's best response into the objective function and assuming the induction hypothesis above, we can re-write the seller problem as choosing the next cutoff:

$$\begin{aligned} V(k, T-t; \Delta) = \max_{k_+} & (1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}) \kappa \left(\frac{k^a - k_+^a}{k^a} \right) \\ & + e^{-r\Delta} \frac{a}{a+1} \gamma_{t+\Delta} \frac{k_+^{a+1}}{k^a}. \end{aligned}$$

The F.O.C. implies that the unique maximum is attained when

$$((a+1)(1 - e^{-r\Delta}) + e^{-r\Delta} \gamma_{t+\Delta}) k_+^a = (1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}) k^a,$$

which yields the optimal cutoff:

$$\kappa^*(k) = \left(\frac{1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}}{(a+1)(1 - e^{-r\Delta}) + e^{-r\Delta} \gamma_{t+\Delta}} \right)^{\frac{1}{a}} k.$$

Hence,

$$\begin{aligned}
 & V(k, T - t; \Delta) \\
 &= \underbrace{(1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}) \left(\frac{1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}}{(a+1)(1 - e^{-r\Delta}) + e^{-r\Delta} \gamma_{t+\Delta}} \right)^{\frac{1}{a}} \frac{a}{a+1} k}_{\equiv \gamma_t} \\
 &= \frac{a}{a+1} \gamma_t k.
 \end{aligned}$$

Finally, the equilibrium price at t has to satisfy the buyer's best response condition (A1):

$$\begin{aligned}
 P(k, T - t; \Delta) &= \kappa^*(k) - e^{-r\Delta}(\kappa^*(k) - \gamma_{t+\Delta} \kappa^*(k)) \\
 &= \kappa^*(k)(1 - e^{-r\Delta} + e^{-r\Delta} \gamma_{t+\Delta}) = \gamma_t k,
 \end{aligned}$$

which completes the proof.

The proposition does not specify the buyer's strategy, but it can be derived from the seller's strategy. Given any price p at time t (on or off the equilibrium path), the cutoff type accepting this price is the unique solution to

$$k - p = e^{-r\Delta}(k - \gamma_t k)$$

$$\text{so that } \kappa(p, T - t; \Delta) = \frac{p}{1 - e^{-r\Delta} + e^{-r\Delta} \gamma_t}.$$

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